B.Sc. part – I (Sub) Set Theory. 07-07-2020.

Q.1. Define an equivalence relation with an example.

Soln: Equivalence Relations:-

Let R be a relation in a set A. Then R is an equivalence relation in A iff

(I) R is reflexive i.e. xRx, $\forall x \in A$.

(II) R is symmetric i.e. $xRy \Rightarrow yRx$, $\forall x, y \in A$

(III) R is transitive i.e. $xRy, yRz \Rightarrow xRz$, $\forall x, y, z \in A$.

For example:

Let A be the set of all real numbers. Let R be the relation in A defined by 'x is equal to y', $x, y \in A$.

Then, we have

(I) $x = x$, $\forall x \in A$, Thus R is reflexive

(II) $x = y \Rightarrow y = x$, $\forall x, y \in A$. Thus R is symmetric.

(III) $x = y$, $y = z \Rightarrow x = z$, $\forall x, y, z \in A$. Thus, R is transitive.

Since, R is reflexive, symmetric and transitive therefore, R is an equivalence relation.

Q.2. Let A be the set of triangles in a plane and a relation R is defined by 'x is similar to y' $x, y \in A$.

Soln: Let R be a relation in a set A. Then R is an equivalence relation in A iff

(I) xRx, $\forall x \in T$. Since every triangle is congruent to itself. Thus R is reflexive.

(II) $xRy = yRx$, $\forall x, y \in T$, since if triangle x is congruent to triangle y then, y is congruent to x. Thus R is symmetric.

B.Sc. Part-I (Sub) Set Theory. 07-07-2020.

Equivalence Relation.

Rest part of Q.2.

(iii) xRy, $yRz \Rightarrow xRz$, $\forall x, y, z \in T$. Since triangle x is congruent to y and triangle y is congruent to triangle z then triangle x is congruent to triangle z. Thus R is triangle.

Since relation R is reflexive, symmetric and ~~...~~ transitive, therefore R is an equivalence relation.

Q.3. Let I be the set of all integers. Let us define a relation R in I as '$x-y$ is divisible by 5' $x, y \in I$.

Soln: Since I be the set of all integers. A relation R in I as '$x-y$ is divisible by 5' $x, y \in I$

Thus, we have

(I) $\forall x \in I$, $x-x=0$ and 0 is divisible by 5.

Thus, R is reflexive.

(II) Let xRy, then $x-y$ is divisible by 5, and hence $y-x = -(x-y)$ is also divisible by 5

i.e. $xRy \Rightarrow yRx$. Thus R is symmetric.

(III) Let xRy, yRz ~~...~~ and $x-y$ and $y-z$ are both divisible by 5. Hence, 5 is also a divisor of $(x-y)+(y-z)$ i.e. $x-z$.

Hence, xRy, $yRz \Rightarrow xRz$.

Since, R is reflexive, symmetric and transitive. Therefore, R is an equivalence relation.

Solved.